Surrogate analysis of coherent multichannel data

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We present a method for generating surrogate data for multichannel time series. By preserving both the power spectra and the cross spectrum of the original data, we can determine if a given statistical test (such as the synchronization index) is being biased by the presence of coherence due to linear superposition of the separate measurements. Current methods based on phase randomization techniques are unsuitable for this particular task. This method is demonstrated on various canonical systems and numerical models. We will show that this algorithm is capable of properly preserving the power spectra and coherence function of the original data, and furthermore, that with the help of surrogate analysis the synchronization index measure is capable of distinguishing between coupled nonlinear oscillators and coherent superpositions of independent chaotic oscillators.

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I. INTRODUCTION

Recent advances in experimental multichannel recording techniques in medicine and neuroscience have stimulated growing interest in synchronization [1–4]. Recent studies in the visual system of cats [1], and the olfactory system of locusts [5,6], have revealed the impact of synchronization on operational performance of sensory systems. Moreover, the application of synchronization ideas to magnetoencephalography (MEG) of Parkinsonian patients has shown the advantage of the synchronization approach in comparison with the traditional cross-correlation (coherence) analysis [7,8]. In particular, it was shown that synchronization measures based on statistical properties of instantaneous phase differences between two channels, allow a more precise localization of the tremor related brain activity [7].

Synchronization is fundamentally a nonlinear effect and is manifested as the adjustment of frequencies or phases of weakly interacting oscillatory nonlinear elements. When two oscillators are synchronized it is expected that they will exhibit a high degree of coherence. On the other hand, coherence of two signals can simply be due to linear superposition, and it must be established that the synchronization measure is not being biased.

An example of this problem can be seen in the synchronization tomography technique developed by Tass *et al.* [7,8]. Current source densities in the human brain are calculated from MEG signals, using a technique known as magnetic field tomography (MFT) [9]. Synchronization analysis can then be applied to pairs of source locations in the brain. The nature of the way the MEG readings are recorded guarantees that each signal has a certain amount of cross contamination from other locations [7]. Methods for distinguishing between synchronization and simple linear coherence are therefore of great importance and practical relevance [10].

The straightforward experimental manipulations which modify system parameters are, in principle, the best way to distinguish between synchronization and linear coherence. For example, application of pharmacological agents can modify coupling strength between neurons, showing changes in dynamics [5,6]. In other cases, changing the natural frequencies of interacting subsystems may reveal the existence of coupling and synchronization [11,12]. Unfortunately, in many cases such experimental procedures are either very difficult to do, or completely impossible as in the case with MEG. Therefore, the statistical methods for generating surrogates and rejecting a null hypothesis must be applied.

The crucial question that arises is: "What effect does the coherence between two signals have on the synchronization measures?" In order to establish whether a given synchronization measure calculation indicates a real dynamical coupling between the signals, or is simply due to linear superposition, statistical significance testing must be done [13].

Surrogate analysis is often used to empirically estimate the distribution of statistical measures. For the purposes required here, the surrogate data should preserve both the power spectra of the two time series being analyzed, as well as the coherence function. Methods for generating surrogates with these properties have been developed [14,15], based on the well-known phase randomized surrogate techniques [16,17]. These methods work by adding random numbers (from 0 to 2π) to the phase of each frequency component of the Fourier transform of the data. The same set of random phases is added to each time series, thereby preserving the phase differences, and thus the coherence function of the original data. This technique is not appropriate for the analysis proposed here, although the reason is somewhat subtle.

The phase differences that would be expected if linear superposition were the source of the coherence will have a specific distribution. Of course, if the phase differences of the original data clearly do not match the expected distribution, we can reject the null hypothesis immediately. But in many cases the distribution of phase differences may not be immediately distinguishable from the expected distribution. Nevertheless, any anomalies that may exist will be exactly preserved by the surrogates described above, which can result in a significant bias. Put simply, the coherence function

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is an averaged quantity that depends on the phase differences. By preserving the exact phase differences from the original data, we preserve not just the coherence function, but also any other correlations that may be encoded by the phase differences. These other correlations could effect the synchronization measures in any number of unpredictable ways.

In this paper we propose a method of surrogate analysis for multichannel data sets that is based on the digitally filtered shuffled (DFS) surrogate technique recently described in Ref. [18]. This method was developed to address a problem with spectral variability in the phase randomized surrogate techniques [15,19]. This paper is organized as follows. In Sec. II we describe the synchronization index and the coherence function. We present the algorithm for generating the coherent digitally filtered (CDF) surrogates in Sec. III. In Sec. IV we demonstrate the technique on some canonical examples.

II. SYNCHRONIZATION INDEX AND COHERENCE FUNCTION

The coherence function measures linear correlations between two stochastic processes $x(t)$ and $y(t)$ in the frequency domain [20], and is calculated as

$$C(f)=\frac{|F_{xy}(f)|}{\sqrt{F_{xx}(f)F_{yy}(f)}}, \tag{1}$$

where the power spectra and cross spectrum are given by

$$F_{xx}(f)=\langle X(f)X^{*}(f)\rangle,$$
$$F_{yy}(f)=\langle Y(f)Y^{*}(f)\rangle, \tag{2}$$
$$F_{xy}(f)=\langle Y(f)X^{*}(f)\rangle,$$

and $X(f)$ and $Y(f)$ are the Fourier transforms of $x(t)$ and $y(t)$, respectively. The indicated averaging $\langle\cdot\rangle$ is understood to be an ensemble average over many realizations of the processes. Of course, in practice these averages must be estimated from the single pair of measured time series. The coherence function ranges from 0 (no cross correlation) to 1 (full coherence).

Phase synchronization between two processes $x(t)$ and $y(t)$ is estimated through the calculation of the differences $\psi_{xu}(t)$ between their instantaneous phases $\phi_x(t)$ and $\phi_y(t)$. See Refs. [10,21] for different approaches to the calculation of the instantaneous phase:

$$\psi_{xy}(t)=m\phi_x(t)-n\phi_y(t), \tag{3}$$

where m and n are integer numbers. In this paper we consider the simplest case of 1:1 synchronization when $m=n=1$. Due to noise in the system, the phase difference diffuses over a periodic potential $U(\psi_{xy})=U(\psi_{xy}+2\pi)$ [22]. The Brownian motion of the phase difference of stochastically synchronized systems contains the phase-locked epochs when the phase difference stays in a potential well of $U(\psi_{xy})$, and phase slips when the phase difference rapidly jumps from one potential well to another. Several measures have been proposed to characterize such stochastic dynamics [10,23]. In the case of the 1:1 synchronization considered here, these measures can be derived from the probability density of the phase difference $P(\psi)$. The uniform distribution corresponds to the absence of synchronization, while a well expressed peak in $P(\psi)$ indicates synchronization. The synchronization index can thus be derived as the first Fourier harmonic of $P(\psi)$ [10]:

$$\gamma^2=\langle\cos\psi\rangle^2+\langle\sin\psi\rangle^2. \tag{4}$$

The synchronization index changes from 0 (uniform distribution of the phase difference, no synchronization) to 1 (δ distribution, strongest synchronization).

III. COHERENT DIGITALLY FILTERED SURROGATES

In Ref. [18] a technique was presented for generating surrogate data that preserves the power spectrum of the original data, but which is consistent with the null hypothesis of a linear stochastic process. This method is based on the basic principles of linear signal processing and can be adapted for the problem at hand.

The first part of the analysis consists of extracting the needed information from the original data. The power spectra F_{xx} and F_{yy}, as well as the cross spectrum F_{xy}, are calculated. This is done by demeaning the data, and then using 50% overlapping samples of length N, where $N/2+1$ is the desired frequency resolution. Fourier transforms of these samples are made using a windowing function, such as the Welch window [24]. The averaging called for in Eq. (2) is then made over these samples. A detailed description of this process can be found in Ref. [25].

The sample size N should be chosen small enough to provide effective averaging, but large enough to resolve any significant structure. Linear filters can be made from the power-spectra estimates of $x(t)$ and $y(t)$ as described in Ref. [18]. That is, by taking the square root of the power estimate, and then using linear interpolation to increase the frequency resolution from $N/2+1$ back to $L/2+1$, where L is the file length. This produces a transfer function that, when multiplied by the Fourier transform of a length L sequence of Gaussian distributed, δ-correlated numbers (white noise), will result in a time series with the appropriate power spectrum.

To reproduce the appropriate coherence function for our surrogates, we look at a simple model for linear superposition. Consider two statistically independent linear stochastic processes $u(t)$ and $v(t)$. We can then make two coherent time series as follows:

$$s_x(t)=\alpha u(t)+(1-\alpha)v(t),$$
$$s_y(t)=(1-\alpha)u(t)+\alpha v(t), \tag{5}$$

where α is a constant from 0 (no coherence) to 0.5 (full coherence). The coherence can easily be calculated for the time series $s_x(t)$ and $s_y(t)$:

$$C = \frac{2\alpha(1-\alpha)}{\alpha^2 + (1-\alpha)^2}. \quad (6)$$

Note that when we write it this way, we see that α can be an arbitrary function of frequency *f*. By doing the sum in Eq. (5) in the frequency domain, we can achieve any coherence function $C(f)$ we want:

$$C(f) = \frac{2\alpha(f)[1-\alpha(f)]}{\alpha^2(f) + [1-\alpha(f)]^2}. \quad (7)$$

One very convenient aspect of the coherence function is that if one (or both) of the time series are passed through a linear filter, the coherence function is not altered. This means that we can generate surrogates of our original time series $x(t)$ and $y(t)$ by making $u(t)$ and $v(t)$ independent white noise, and then use an appropriate function $\alpha(f)$ to get s_x and s_y, which can then be filtered to get the appropriate power spectra.

To select the proper values of α (as a function of frequency), we simply invert Eq. (7),

$$\alpha(f) = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{C(f)}{2[1+C(f)]}}. \quad (8)$$

It should be noted that the power spectra of s_x and s_y will no longer be white, due to the summing above. The filters must therefore be altered to compensate for this effect. The power spectra of both s_x and s_y are now given by

$$F_{s_x s_x}(f) = F_{s_y s_y}(f) = \alpha^2(f) + [1-\alpha(f)]^2. \quad (9)$$

The filters can easily be adjusted by simply dividing the transfer function (the filter in the frequency domain) by the square root of Eq. (9). By using different realizations of Gaussian white noise for each surrogate pair, we can generate a population of surrogate pairs s_x and s_y, that each have the same power spectra and coherence function as the original data *x* and *y*.

The distribution of the surrogate time series will be Gaussian, with zero mean and a variance which depends on the normalization properties of the discrete Fourier transform method used. If the distributions of the original time series are also Gaussian, then rescaling the distributions of the surrogates to that of the original data is trivially simple:

$$S_x(t) = s_x(t)\frac{\sigma_x}{\sigma_{s_x}} + \langle x(t)\rangle,$$

$$S_y(t) = s_y(t)\frac{\sigma_y}{\sigma_{s_y}} + \langle y(t)\rangle, \quad (10)$$

where σ_x and σ_{s_x} indicate the standard deviation of $x(t)$ and $s_x(t)$, respectively. Likewise for $y(t)$ and $s_y(t)$. The time series $S_x(t)$ and $S_y(t)$ are the final, rescaled surrogates. If the distribution of the original data is not Gaussian, then the rescaling becomes more complicated. The iteration scheme for multivariate phase randomized surrogates presented by Schreiber and Schmitz [15] can be applied here as well, although no investigation has been done yet to determine the criteria for convergence. For the purposes of this paper, we shall limit our discussion to cases where the distributions of the data are approximately Gaussian.

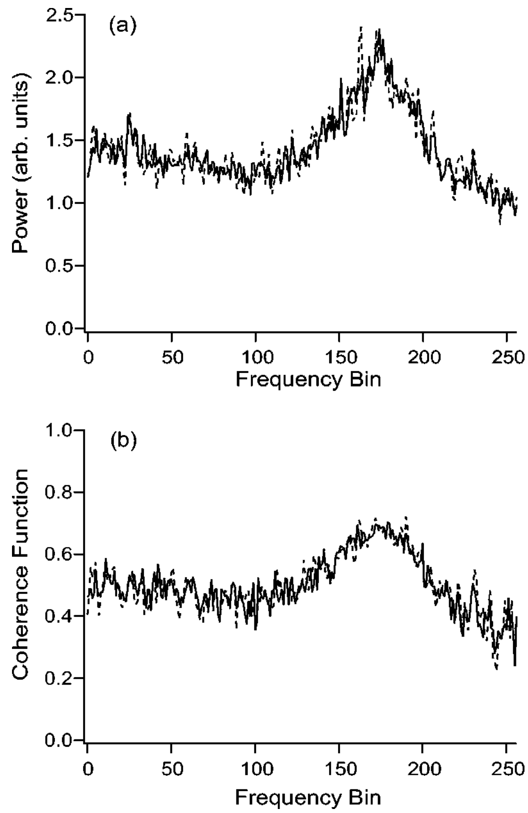

FIG. 1. (a) Power spectra of the original time series $x(t)$ (solid line), and a surrogate $S_x(t)$ (dotted line). (b) Coherence function of the original AR data (solid line), and the surrogate data (dotted line). 256 frequency bins were used.

IV. CANONICAL EXAMPLES

To demonstrate that the algorithm described here actually does preserve the power spectra and coherence function of the input time series, we test it on a simple linear stochastic process. Consider the following autoregression (AR) process

$$v_n = -0.1v_{n-2} + 0.2v_{n-3} - 0.1v_{n-4} + \xi_n, \quad (11)$$

where ξ_n is Gaussian white noise with zero mean and unit variance. The coefficients for this process are chosen arbitrarily, so as to produce a power spectrum that has a broad, distinctive peak, as can be seen in Fig. 1(a). We also generate two time series of statistically independent Gaussian white noise, $u_1(t)$ and $u_2(t)$, with zero mean and unit variance. We then generate two time series of coherent data as follows:

$$x(t) = v(t) + u_1(t),$$
$$y(t) = v(t) + v_2(t). \quad (12)$$

We generate $x(t)$ and $y(t)$ with 65 536 points each. We also generate surrogates using the CDF method described in Sec. III. In Fig. 1(a), the power spectra of both the time series $x(t)$ and its surrogate $S_x(t)$ are shown. Note that both $x(t)$ and $y(t)$ have the same power spectra by construction. In Fig. 1(b), the coherence function is shown for both the original data and the surrogate data. We can see from these figures that both the power spectra and coherence function of the original data have been preserved.

To demonstrate this surrogate technique in an actual phase synchronization test, we present two coupled Lorenz oscillators [26–28]:

$$\dot{x}_{1,2} = 10(y_{1,2} - x_{1,2}) + \epsilon(x_{2,1} - x_{1,2}) + \sqrt{2D}\xi_{1,2}(t),$$
$$\dot{y}_{1,2} = 28x_{1,2} - y_{1,2} - x_{1,2}z_{1,2} \tag{13}$$
$$\dot{z}_{1,2} = -\tfrac{8}{3}z_{1,2} + x_{1,2}y_{1,2},$$

where ϵ is the coupling strength. Dynamical noise was added with an intensity of $D = 0.1$, where $\xi_{1,2}$ are statistically independent white noise with zero mean and unit variance. We have chosen a coupling strength of $\epsilon = 3.0$. The other parameters were chosen to produce chaotic behavior. Equation (13) was numerically integrated with a time step of 0.01, for a total of 524 288 time steps. To calculate the instantaneous phases of the Lorenz systems we introduce two new variables [10,21]:

$$A_{1,2}(t) = \sqrt{x_{1,2}^2(t) + y_{1,2}^2(t)}, \tag{14}$$

and then calculate the phases $\phi_{1,2}(t)$ and the phase difference $\phi_1(t) - \phi_2(t)$ using the analytic signal approach applied to $A_{1,2}(t)$ [7]. The same time series of $A_{1,2}(t)$ were used to calculate the power spectra and coherence function. Figure 2 shows the power spectrum and coherence function for this data, as well as for a surrogate pair. We see that, once again, the surrogate data preserves the power spectra and the coherence function of the original data.

The synchronization index was calculated for both the original data and 19 realizations of the surrogate process. The distribution of instantaneous phase difference for the Lorenz data is shown in Fig. 3(a). The distribution of instantaneous phase difference for one of the surrogate pairs is shown in Fig. 3(b). For the original data we get $\gamma_A = 0.583$, which exceeds the maximum for all of the surrogates of $\gamma_{S_{\max}} = 0.470$. Since Gaussian statistics cannot be assumed, we apply a nonparametric rank-order statistic [16]. If we have N surrogates, then the probability of a false rejection of the null hypothesis is $1/(N+1)$. We can, therefore, reject the null hypothesis at the 95% confidence level.

As a control test, we have generated data from two uncoupled Lorenz oscillators ($\epsilon = 0$). Linear superpositions of these time series were then made using the coherence function estimated from the coupled Lorenz systems. This was done by summing the Fourier transforms of the uncoupled oscillators $U(f)$ and $V(f)$ as follows:

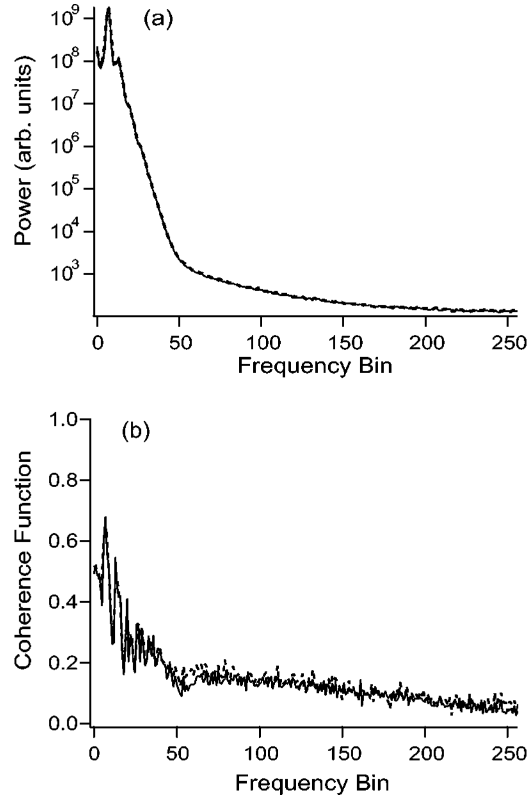

FIG. 2. (a) Power spectra of the original time series $A_1(t)$ (solid line), and a surrogate $S_{A_1}(t)$ (dotted line). Note that a log scale is used due to the strongly peaked spectra. (b) Coherence function of the original Lorenz data (solid line), and the surrogate data (dotted line). 256 frequency bins were used.

$$X_1(f) = \alpha(f)U(f) + [1 - \alpha(f)]V(f),$$
$$X_2(f) = [1 - \alpha(f)]U(f) + \alpha(f)V(f), \tag{15}$$

where $X_1(f)$ and $X_2(f)$ are the Fourier transforms of the superposed oscillators $A_1(t)$ and $A_2(t)$, respectively. The function $\alpha(f)$ was calculated from the coherence function estimated from the coupled Lorenz oscillators, using Eq. (8). The synchronization index was then calculated for the superposed data, as well as for 19 surrogate pairs. The distribution of instantaneous phase difference for both the superposed Lorenz data and one of the surrogate pairs is shown in Fig. 4.

In this case, we see that the superposition of Lorenz oscillators actually gives a synchronization index slightly lower than that of the surrogates, indicating that the high degree of synchronization found in the coupled data is due only to the nonlinear correlations between the two oscillators, and not to the nonlinear behavior of the individual oscillators.

Similar results were also obtained using a pair of noisy coupled Rössler oscillators [29] with the parameter values described in Ref. [30]. However, an extremely high-frequency resolution is required to generate proper surrogates for this system because of the very sharp peak obtained for the power spectrum. Recall that the sample size used for the calculation of the power spectrum and coherence func-

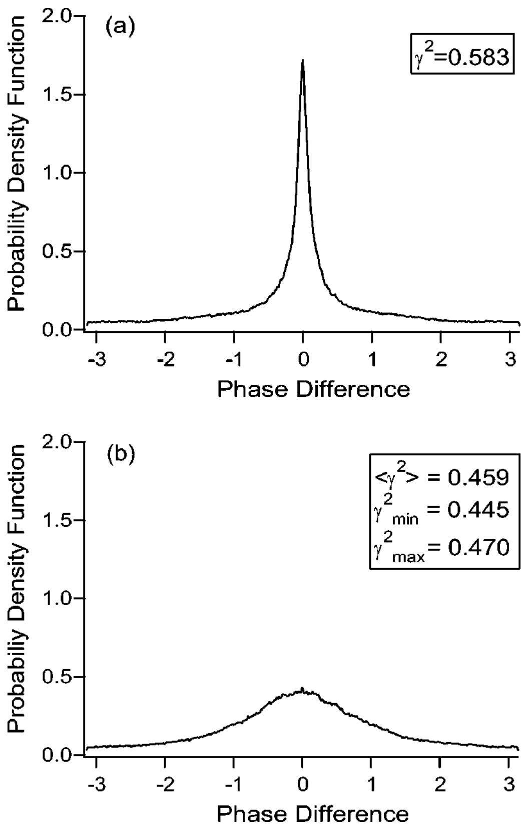

FIG. 3. Distribution of instantaneous phase difference for the coupled Lorenz data (a), and for the surrogate pair giving the maximal value of the synchronization index (b). In both cases the synchronization index is shown in the inset.

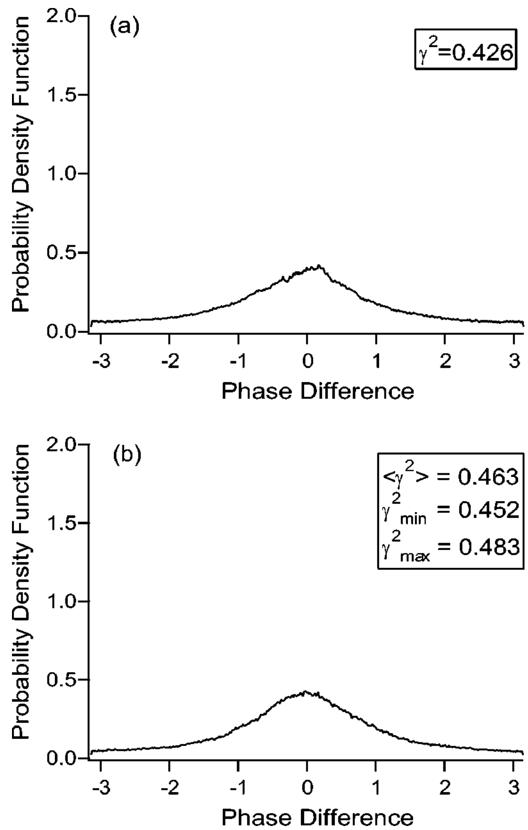

FIG. 4. Distribution of instantaneous phase difference for the linear superposition of uncoupled Lorenz oscillators (a), and for the surrogate pair giving the most typical value of the synchronization index (b). In both cases the synchronization index is shown in the inset.

tion must be small enough to allow for sufficient averaging in Eq. (2), but large enough to accurately resolve any spectral peaks. Thus this method may not be suitable for data which, like the Rössler system, has a very sharp spectral peak, unless very long data sets are available.

V. DISCUSSION

The surrogate analysis method described here tests the null hypothesis that the two time series being analyzed are superpositions of linear stochastic processes. We have shown that the synchronization index gives significantly different results for actual phase synchronized, coupled Lorenz oscillators than for linear stochastic data with the same coherence function and power spectra. Furthermore, by showing that the synchronization index gives similar results for linear superpositions of independent Lorenz oscillators and linear superpositions of linear stochastic data, we have demonstrated that the nonlinear statistical properties of the individual time series are not responsible for the results.

In general, a rejection of the null hypothesis does not conclusively prove that nonlinear dynamical coupling is present. The purpose of this technique is not to prove that such coupling is present, but rather to demonstrate that a model of a linear superposition of independent oscillators is not sufficient to describe the data. Furthermore, this technique provides a method for establishing what value of the synchronization index is expected for a given degree of coherence. This is very important in the analysis of spatiotemporal data where both the coherence and synchronization index vary as a function of position. By "normalizing" the synchronization index to the expectation value estimated by the coherence function, the ability to identify regions of high synchronization can be much improved.

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